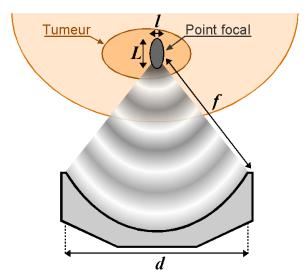
How to use ARFI to refocus the HIFU beam?

Before stating the actual problem, we describe the HIFU beam:

A High Intensity Focused Ultrasound (HIFU) transducer usually has a spherical shape in order to generate at the focal point a constructive interference of ultrasound waves with the same phase. This constructive interference (similar to a lens and focusing of light) triggers heating at the focal point. While using a single block transducer focusing in homogenous medium the size of the ellipsoidal focal point I×L can be defined as a function of the transducer focal length and aperture diameter d.



As shown on this figure the HIFU transducer is usually placed in front of tumoral tissue to ablate the tumor without carrying out invasive surgery. During this intervention, the MRI can be used to measure the temperature at any location via a method known as PRF (Proton Resonance Frequency) and recently the acoustic intensity distribution can be also measured by MRI using Acoustic Radiation Force Measurements (ARFI). It has been observed that the multiple tissue layers located between the transducer and targeted tumor can significantly (if not completely) defocus the beam, making it especially difficult to target brain tumours (given the rigid skull as well as the intermediate layers between transducer and tumour).

To understand better the physics of HIFU, we describe briefly the simulation of the acoustic field in a homogenous medium:

The acoustic field in a homogenous medium can be simulated by the Rayleigh integral:

$$P_{(X,Y,Z)} = \frac{\rho \cdot V \cdot \omega}{\lambda} \cdot \iint_{S} \frac{\exp(i \cdot \vec{k} \cdot \vec{r})}{\vec{k} \cdot \vec{r}} dS$$

This integral defines the acoustic pressure P in a medium of density ρ for a surface S vibrating with a pulsation frequency ω and a speed V. The pressure obtained at the point (x,y,z) is a complex number which represents the amplitude and the phase of the acoustic field. In this expression \vec{r} is a vector from a point of the transducer surface dS to the simulated pressure point (x,y,z). The wave vector \vec{k} has a modulus equal to 2π divided by the known wave length λ :

$$\left| \vec{k} \right| = \frac{2\pi}{\lambda}$$

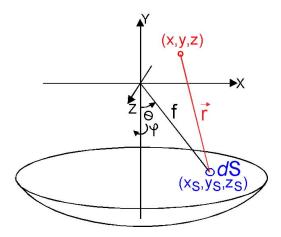
The orientation of the vector \vec{k} is defined by the orientation of the ultrasound wave propagation and can be approximated to be aligned with the vector \vec{r} .

$$\vec{k} \cdot \vec{r} = |\vec{k}| |\vec{r}|$$

The length of the vector \vec{r} is defined by the distance between the transducer surface point (x_s, y_s, z_s) and the simulated acoustic filed point (x,y,z).

$$|\vec{r}| = \sqrt{(X - X_S)^2 + (Y - Y_S)^2 + (Z - Z_S)^2}$$

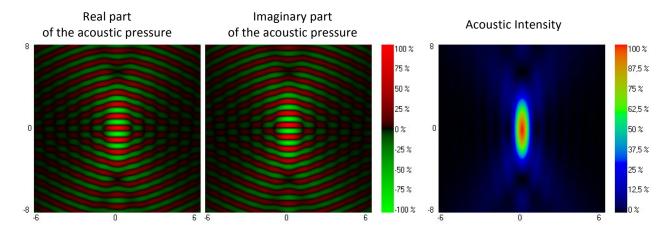
Since the transducer has a spherical shape this integral can be numerically processed using spherical coordinate (θ, ϕ, f) to express the location of each elementary point dS.



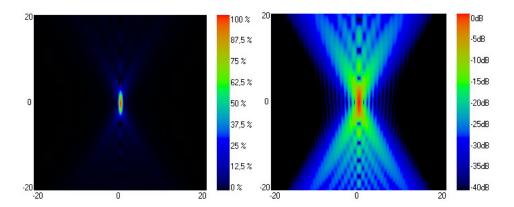
The resulting acoustic field is proportional to the square of the modulus of the acoustic pressure:

$$I_{(x,y,z)} \propto \left| P_{(x,y,z)} \right|^2$$

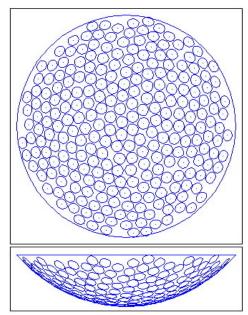
This figure presents an example of acoustic field produced by a single block transducer with a focal length of 80mm and an aperture diameter of 96mm:



The distribution of the acoustic field intensity can be displayed using either a linear or a logarithmic scale to visualize fine details:

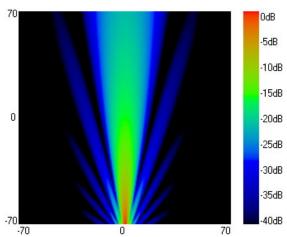


To be able to control the location of the focal point or the shape of the beam, modern HIFU transducers (named phased array transducers) are composed of multiple small transducers (known as elements) with individual control of the amplitude and phase of the ultrasound signal emitted by each element. Those elements are distributed over a spherical surface in a compact and asymmetrical pattern in order to reproduce a field similar to a large single block transducer.



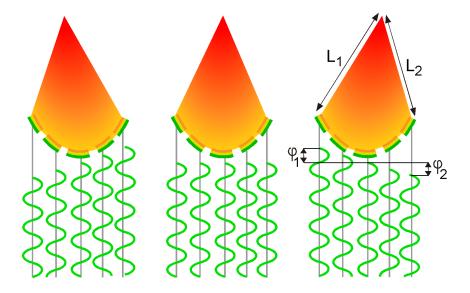
Asymmetrical and compact phased array transducer composed of 256 elements

The acoustic field of each element can be also processed using the Rayleigh integral in the same way as for a large single block transducer. This figure shows an example acoustic field distribution generated by one of these elements with a focal length of 80mm and 5.8mm aperture diameter



Intensity distribution generated by a single element with a focal length of 80mm and 5.8mm aperture diameter

Due to the low aperture of each element the resulting intensity distribution is a diverging beam due to diffraction effects. However, the sum of the pressure of each element induces a focused intensity distribution and hence a focused beam similar to the one shown for a large single block transducer. The main application of phased array transducers is the electronic displacement of the focal point by adjusting the phase of the electrical signal applied on each element to produce constructive interference at the desired location:



Along the ultrasound propagation orientation the phase changes by 2π for every wave length λ . As a consequence the phase Φ n of the electrical signal of the element n can be extracted from this phase law:

$$\Phi_n = 2\pi \frac{L_n - L_0}{\lambda}$$

Beam refocusing:

Electronic displacement of the focal point is a well controlled and established technique, but using the phase applied on each element to compensate for tissue heterogeneities is a much more complex problem. Simulation of the impact of tissue heterogeneities can be handled by segmenting each tissue and using the Rayleigh integral to process the propagation of the ultrasound from one layer to the next one, assuming that each layer is sufficiently homogenous. But such processing requires a fine knowledge of tissue properties and a high resolution of segmentation for each tissue layer (i.e much smaller than the wave length $\lambda \approx 1 \text{mm}$ because the phase changes by 2π over the course of λ).

The article 'Hertzberg2010' proposes a very interesting alternative approach which aims to measure the phase from each element at the focal point using ARFI instead of simulating each layer. Unfortunately the ARFI provides only quantification of the intensity distribution but no direct measurement of the phase of the pressure distribution. As a work around, all elements are turned to a power level equal to 20W (using about 0.1W per element) except for one element used at the maximum power level of 2W using a different phase. The positive or negative interference of this element in comparison to the others allows identification of the optimal phase to use in order to refocus the beam.

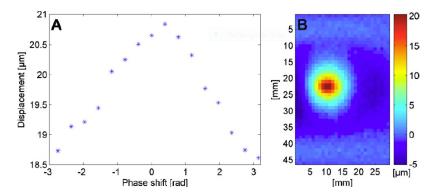


FIG. 1. (a) Maximal displacement response to a single phased array element's phase. MR-ARFI image (b) obtained by the line-scan spin-echo method was used to calculate one of the maximal displacement points presented in the graph. The images were acquired by 21 W of acoustic power transmitted from the 208-element phased array, while the power transmitted from the tested element was increased to 2 W.

However this method requires acquisition of 10 different intensity distributions via the ARFI method to optimize the phase of a single element. The optimization of the phase of a 256 channels transducer would require 256×10 = 2560 ARFI acquisitions. In addition the reference phase generated by all elements is initially not refocused, so that a second iteration is most likely necessary to get optimal refocusing which leads to a total of 5120 ARFI acquisitions.

So many ARFI acquisitions has multiple draw backs, it requires a very long acquisition time, one ARFI image requires typically 1s which leads to a total acquisition time of around 85min. In addition each ARFI image acquisition requires emission of an ultrasound pulse which can be considered negligible for one acquisition but can add up quickly when considering multiple ARFI acquisitions above the ablative threshold when trying to refocus the beam prior to treatment. As consequence an additional temperature cooling waiting period is necessary between each ARFI acquisition, which makes the use of the ARFI refocusing method unusable clinically since the patient can't wait for hours on the MRI bed (especially under anesthesia).

However the ARFI refocusing algorithm can most probably be optimized using a different power than 2W on a tested element and 20W on another element. Multiple elements can be tested simultaneously using either groups of elements using a pattern such as Hadamard (as reported in 'Marsac2012 ') or by dividing the transducer into regions (as reported in 'Hertzberg2010'). Another way to optimize the refocusing algorithm can be the use of more information than the intensity at the targeted voxels. Each ARFI acquisition is a 2D image in which only one point of this image is used. Neighboring voxels contain relevant information which can be also used to improve the refocusing algorithm.

To summarize, the proposed problem concerns refocusing in heterogeneous media:

What is the optimal combination of element signals to test in order to deduce (with the minimal amount of ARFI acquisitions) the phase (and optionally amplitude) to apply on each element to achieve perfect constructive interference at a targeted point?