Discussion Document for Ontario Deans of Engineering on High School Preparation

In this document we put forward a "proposition" which has emerged from several years of rather broad discussion among high school, college and university mathematicians. A strong endorsement of this from Ontario engineering departments could have a significant impact on further curriculum revisions now being contemplated by the Ontario Ministry of Education.

In the mid 1990's a new curriculum policy document was written for Ontario Secondary Schools. A significant feature of this curriculum was a change in the high school experience from 5 years to 4, with the final high school year being grade 12, and thus many, perhaps most, high school graduates of the future would be a year younger. In addition, students who graduated in 4 years would have fewer overall courses, including fewer final year courses.

The three Grade 12 Mathematics courses outlined in this document were:

- Advanced Functions and Introductory Calculus
- Geometry and Discrete Mathematics
- Mathematics of Data Management

The team that wrote the initial drafts of the policy document was based at the Fields Institute for the Mathematical Sciences in Toronto and consisted of participants from university, colleges, schools, school boards and industry. Given that there was to be one less year, the curriculum design team debated whether, and how much, the math curriculum should be cut back. How much of what was currently being taught in school should be moved into college and university? Which material would be moved to the elementary schools? In addition, there was a desire to extend the curriculum, to help develop students' problem-solving, communications skills, general learning skills, with project work, and modeling.

The calculus question

The team felt that these larger goals of problem-solving, inquiry and communication were critical and that to enable them it was essential that the curriculum not be overburdened with too much material. The conclusion it reached was that there should not be a formal calculus course in grade 12, rather the "mainline" grade 12 course (the one taken by most students and required by most university and college programs) could contain a number of the big ideas shared with calculus, such as rate of change, optimization, modeling and working with graphs, but should leave the formal technical manipulations of calculus to be quickly and effectively covered in college or university. Such a "capstone" high school course, so called because it could serve to tie together a number of significant ideas from earlier grades, would also help in the development of skills in problem-solving, communications, and modeling.

Apparently, however, at that time (the late 90's) engineering and science departments were asked for their views on the inclusion of calculus in high school and generally voted yes. It's possible however that if such a survey was taken, the wrong questions were asked, or the options and consequences were not made clear. Anyway, the design team was instructed that the mainline grade 12 course would be calculus—plus, as it turned out, those aspects of exponential and polynomial functions that there was no time to cover in grade 11.

The installation of calculus at the end of this 4-year curriculum put heavy "precalculus" demands on the grade 11 and even on the grade 10 courses. A consequence of this is that a number of important topics fell by the wayside, for example, geometry, discrete mathematics (logic, counting) and probability, not to mention skills of problem-solving, inquiry and modeling. Moreover, all courses from grade 10 through grade 12 calculus became crowded and teachers experienced serious difficulties covering all the material with a conceptual basis, as well as supporting rich problem solving activities.

Proposition. The formal technical treatment of calculus should be removed from the high school curriculum and replaced by a capstone experience consolidating a number of concepts and techniques from earlier grades and providing rich experiences in problem-solving, communication, inquiry-based learning and mathematical modeling.

This would be accompanied by adding trigonometry to grade 12, and reducing the total number of detailed topics now packed into the grade 10 and 11 courses, encouraging conceptual understanding and rich explorations in these grades, building towards the capstone experience.

Our impression from canvassing our colleagues is that almost all math departments in Ontario would support this proposition. If students arrived with good learning skills, and a good conceptual grounding for the larger ideas, the basic mathematical content of Calculus, Linear Algebra, and Discrete Mathematics could be mastered in their first year courses at university or college.

Comparison with other jurisdictions.
In the current Ontario system, calculus is part of the *fourth* high school math course. In all other jurisdictions that we know of which offer calculus at the high school level, it is the *fifth* course in the high school sequence. This might well suggest that the fourth course is too early. We offer two examples.

British Columbia has a four-year high school program similar to ours and the standard Grade 12 "Principles of Mathematics" contains no calculus. Admission to the Engineering program at UBC is made on the basis of this grade 12 course together with Physics, Chemistry and English. Nevertheless some 400 of the 650 students typically admitted each year have AP Calculus (as well as Grade 12 Math). Such students, if they have attained a mark of at least 4, are eligible to skip Math 100 and instead take a special section of the second semester course Math 101. However only about 5% of these 400 students opt to do this.

Alberta has a four-year high school program similar to ours and the standard Grade 12 course Math 30 contains no calculus. There is a regular calculus course offered in most school (Math 31--with Math 30 as a prerequisite) and admission to the Engineering program at the University of Alberta *requires* both of these math courses (together with Physics, Chemistry and English). The University of Calgary requires both Math 30 and 31, but does also offer an alternative stream for students who do not have the Math 31.

The shape of Grade 12 as we'd like to see it.
What mathematics courses might we want in Grade 12 in addition to the standard capstone course described in Proposition 1? The Data Management course seems like a valuable addition. Because of its more elementary nature, many math-oriented students are taking it in grade 11. Some of these students would also like an additional Grade 12 course. No doubt many high schools will continue to offer AP Calculus for such students, but our preference would be to have a course for them to take which does not so much "move ahead" with university material, but offers them an opportunity to take a dozen rich problems in various basic areas and explore them fully with their teacher.

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