THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

CHRIS WOODWARD
Rutgers University

Lowest d in quantum Schubert calculus

We give a combinatorial formula for the lowest power of q appearing in the quantum product of two Schubert classes, using the geometry of Kontsevich's moduli space of stable maps. I'll focus on the Grassmann case, in which case the formula is in terms of combinatorics of Young diagrams. (Joint work with W. Fulton)

PING XU
Pennsylvania State University

Dynamical r-matrices, symplectic fibration and quantization

Using symplectic fibration, we give a geometric construction of non-degenerate triangular dynamical r-matrices. Quantization of a non-degenerate triangular dynamical r- matrix can be reduced to a deformation quantization problem for the corresponding symplectic manifold. In particular, when the base Lie algebra is Abelian, we solved the existence and classification problem for quantizations of a non-degenerate triangular dynamical r-matrix.

STEPHANIE FRANK SINGER
Haverford College

The Kepler Problem

We will discuss the $SO(4)$ symmetry of the classical and quantum two-body problem.

MAXIM BRAVERMAN
Northeastern University

Index theorem for transversally elliptic operators on open manifolds

Let M be a an open manifold without boundary acted upon by a compact connected Lie group G and let D be a G-equivariant Dirac-type operator on M. Let $v : M \to LieG$ be a G-equivariant map such that the corresponding vector field on M does not vanish outside of a compact subset. It was noted by E. Paradan that such a data define an element of compactly supported transversal K-theory. Hence, one can define a topological index by embedding of M into a compact manifold.

We introduce a *deformed Dirac operator* on M, whose index is defined as a distribution on G. Our index theorem states that this analytical index equals the topological index introduced by Paradan. As a main step in the proof we show that the analytical index is invariant under non-compact cobordism of the type considered by Guillemin, Ginzburg and Karshon.

MIKHAIL KOGAN
Northeastern University

Symplectic leaves of the standard Poisson-Lie structure

Using generalized minors, we present an explicit description of all symplectic leaves of the complex simple Poisson-Lie group G with the standard Poisson structure. This allows us to compute the number of connected components in the orbit of the dressing action of the dual Poisson-Lie group G^* on G. This is a joint work with A. Zelevinsky.

MIGUEL ABREU
The Fields Institute and Instituto Superior Tecnico

Kähler geometry on toric orbifolds and $U(n)$- invariant extremal metrics

In the first part of this talk I will describe a symplectic approach to Kähler geometry on toric orbifolds. The main result is an effective parametrization of all toric Kähler metrics via smooth functions on the moment polytope.

In the second part of the talk I will give an application to the description of $U(n)$-invariant extremal Kähler metrics. Through this symplectic approach these arise naturally as solutions to a linear second order ODE.

JOHN HARNAD
Concordia University

Multi-Hamiltonian structures, classical r- matrix systems, spectral transforms and separation of variables

Classical r-matrices of rational, trigonometric and elliptic type may be used to define polynomial families of multi-Hamiltonian structures on loop algebras and loop groups corresponding, respectively, to linear (Lie-Poisson) and quadratic (Poisson-Lie/Sklyanin) brackets. The standard r-matrix theory implies commutativity and complete integrability of the flows generated by spectral invariants on finite dimensional Poisson submanifolds consisting of meromorphic Lax matrices over the base curve (of genus 0 or 1) with given pole divisor. The spectral transform yields an identification with the space consisting of pairs of: spectral curves and sheaves supported on them, on which a natural family of algebro-geometric Poisson structures is defined.

A third way to view such Poisson spaces is by identification with symmetric products of a holomorphic Poisson surface with itself, leading to a separation of variables of the flows generated by spectral invariants in the associated "spectral Darboux coordinates". It is shown that the generalized Gel'fand-Zakharevich commuting invariants associated to the multi-Hamiltonian structures are the same as the spectral invariants derived from the r-matrix theory, and the "Nijenhuis-Darboux" coordinates given by the eigenvalues and eigenvectors of the Nijenhuis tensor coincide with the spectral Darboux coordinates. Examples illustrating these results include: constrained oscillators in n-dimensions, classical spin systems, Toda and Volterra lattices and reduced systems associated with commuting flows on the stationary manifolds of the NLS, KdV and Boussinesq hierarchies.

SIYE WU
University of Adelaide and University of Colorado

Projective flatness in geometric quantization

In geometric quantization, the quantum Hilbert space depends not only on the symplectic manifold, but on additional data such as the prequantum line bundle and polarization. These Hilbert spaces can be identified if there is a projectively flat connection on the space of polarizations considered. A well-known example is the quantization of the Chern-Simons gauge theory, in which case there is a projectively flat connection on the moduli space of complex structures on a Riemann surface. In this talk, we discuss the projective flatness in geometric quantization, especially when the symplectic manifold is a linear space.

SAM EVENS
University of Notre Dame

Poisson structures on orbits and compactifications

We show some Poisson structures related to the Bruhat-Poisson structure on a conjugacy class in a complex semisimple Lie group, and on the DeConcini-Procesi compactification of a complex symmetric space. (This is joint work with Jiang-Hua Lu).

YAEL KARSHON
Hebrew University

Quantization of toric varieties

In a Hamiltonian torus action with isolated fixed points, every reduced space is cobordant to a disjoint union of toric varieties. These toric varieties carry a *stable complex* structure, incompatible with their symplectic structure. For a *Kähler* toric variety corresponding to a convex polytope Δ it is known that the quantization corresponds to the lattice points in Δ. We generalize this result to the stable complex case.

VICTOR GUILLEMIN
Massachusetts Institute of Technology

Equivariant Morse theory and graphs

Let G be an n-torus, M a G-manifold and f a G-invariant Morse function if all the critical points are of even index. the stable and unstable manifolds of f support equivariant cohomology classes, and the restrictions of these classes to a given fixed point, p, is a family of polynomials. We discuss some recent results (with Catalin Zara) on the computation of these polynomials.

CATALIN ZARA
Yale University

Generators in equivariant cohomology and equivariant K-theory

For manifolds with a Hamiltonian torus action, generators in equivariant cohomology can be expressed as "path integrals". For Grassmannians of $k-$planes in $\mathbb{C}^n$, these spaces of paths are non-singular Schubert varieties, and we give a similar description for generators in equivariant K-theory.

EYAL MARKMAN
University of Massachusetts

The Classical Dynamical Yang-Baxter Equation;
A geometric interpretation

The Classical Yang Baxter Equation (CYBE) is an algebraic equation for a meromorphic function with values in the tensor square of a Lie algebra. The equation is central in the theory of integrable systems. Drinfeld, following work of Sklyanin, discovered the geometric meaning of the CYBE: It corresponds to a Poisson structure on the loop group. Quantization of the CYBE led to the theory of Quantum groups. Felder (1994) discovered a differential equation (CDYBE), which is a generalization of the CYBE arising naturally in Conformal Field Theory.

We will review work of Etingof and Varchenko on the geometric interpretation of the CDYBE in terms of Poisson groupoids. We will then provide a simple explanation of the geometry in terms of moduli spaces of principal G-bundles on an elliptic curve, for any reductive group G (Joint work with J. Hurtubise).

BERTRAM KOSTANT
Massachusetts Institute of Technology

Dirac cohomology for the cubic Dirac operator

If $\mathfrak{r}$ is a symmetric Lie subalgebra (i.e. fixed under an involution) of a semisimple Lie algebra $\mathfrak{g}$ and V is a $\mathfrak{g}$-module then Vogan has introduced the notion of Dirac cohomology $H_D(V)$. He has also made conjectures about the action of $Z(\mathfrak{g})$ (the center of $U(\mathfrak{g})$) on $H_D(V)$. These conjectures have been proved by Huang and Pandzic in a paper to appear in JAMS. Using the the cubic Dirac operator, which we have introduced for other purposes, we extend the notion of Dirac cohomology and show the results of Huang and Pandzic extend to the case where $\mathfrak{r}$ is any reductive Lie subalgebra of $\mathfrak{g}$ so long as $B_{\mathfrak{g}}|\mathfrak{r}$ is non-singular. Here $B_{\mathfrak{g}}$ is a symmetric ad-invariant bilinear form on $\mathfrak{g}$. Involved is the determination of a homomorphism $\eta : Z(\mathfrak{g}) \to Z(\mathfrak{r})$. The determination of η depends upon showing the existence of sufficiently many modules V for which $H_D(V) \neq 0$. The homomorphism η is such that, using a famous result of H.Cartan,

$$H^*(G/R, \mathbb{C}) = Tor^{Z(\mathfrak{g})}(\mathbb{C}, Z(\mathfrak{r})$$

RON DONAGI
University of Pennsylvania

Gerbes, genus-1 fibrations, dualities, integrable systems, and mirror symmetry

The moduli space of vector bundles on a variety X which admits an elliptic fibration $f : X \to B$ (with a section $\sigma : B \to X$) can be described in terms of data on the base B via the Fourier-Mukai transform, or the spectral construction. This is an "elliptic" version of the various integrable systems of meromorphic Higgs fields on B: instead of Higgs fields with values in the canonical bundle (or any other vector bundle) on B, one considers Higgs fields with values in the family f of elliptic curves.

The extension of this result to the case where $f : X \to B$ is a genus 1 fibration (having no section $\sigma : B \to X$) leads to some surprising new features, including the appearance of gerbes, or non- commutative structures, on X. In a sense, the non-commutativity of such a structure is dual to the non-existence of a section. I will formulate a still partly conjectural duality of derived categories on these gerbes, and will show how it incorporates the Fourier-Mukai transform and other known results.

Mirror symmetry between Calabi-Yau threefolds is conjectured (by Strominger, Yau and Zaslow) to be realized geometrically by a duality quite similar to Fourier-Mukai, but involving special Lagrangian tori instead of elliptic curves. The CY analogue of our results suggests a modification and strengthening of the SYZ conjecture. In particular, there should be a real integrable system on the stringy moduli space of Calabi-Yaus (i.e. the moduli of the complex structure, Kahler structure, and B-field) on which mirror symmetry acts. (Joint with Tony Pantev).

ANDREW D. HWANG
College of the Holy Cross

Extremal Kähler Metrics and the Momentum Construction

Associated to a Kähler form ω on a compact Kählerian manifold N is its *Calabi energy*—the L^2-norm of its scalar curvature, computed with respect to the natural volume form. It is of geometric interest to find Kähler metrics that are critical for the Calabi energy among representatives of a fixed de Rham class. Critical metrics, which are necessarily minima, are called *extremal Kähler metrics*.

In this talk, I will survey known facts about extremal metrics, then describe how the momentum function associated to a circle action can be used to construct extremal metrics on certain manifolds obtained by compactifying suitable holomorphic line bundles. There is a necessary and sufficient *complex-analytic* criterion for these spaces to admit metrics of constant scalar curvature. This condition can be phrased in terms of a moment polytope, which yields nice insights into the geometry of these metrics.

ANDRAS SZENES
Massachusetts Institute of Technology

Trace functional on the quantized moduli space of flat connections

We give a simple construction of an algebraic non- commutative deformation of the moduli space of flat connections on a Riemann surface over a ring of rational functions in a parameter q. We show that there is a natural trace functional on this algebra which has values in such holomorphic functions defined for $0 < |q| < 1$, which have an asymptotic expansion as q approaches 1 along the real line. We give an interpretation of these results in the framework of formal products on symplectic manifolds.

GREG LANDWEBER
MSRI, Berkeley and University of Oregon

Dirac operators for Kac-Moody algebras and homogeneous loop spaces

Recently, Kostant investigated a cubic Dirac operator associated to a pair (g,h), where g is a semi-simple Lie algebra and h is a reductive subalgebra of maximal rank in g. The kernel of this Dirac operator exhibits a generalized form of the Weyl character formula, and it associates to each irreducible g-module a set of h-modules known as an Euler number multiplet.

This talk reformulates Kostant's work in the Kac-Moody setting, replacing g and h with the extended loop algebras Lg and Lh. Here, the Dirac operator lives in a loop group analogue of the non-commutative Weil algebra introduced by Alekseev and Meinrenken. The Dirac operator associated to Lg and the Weil algebra quantization map behave differently for Kac-Moody algebras than in the finite dimensional case due to normal ordering. However, in the relative case for a pair (Lg, Lh), Kostant's results have immediate analogues which give a generalization of the Weyl- Kac character formula.

Geometrically, Kostant's cubic Dirac operator corresponds to a formal version of the Dirac operator on the homogeneous space G/H (although NOT for the Levi-Civita connection). Using the Kac-Moody Dirac operator as a model for the geometric Dirac operator on the homogeneous loop space L(G/H) = (LG)/(LH), we show that its index in the infinite level limit approximates the elliptic genus.

YOUNG-HOON KIEM
Stanford University

Cohomology pairings on singular quotients in geometric invariant theory

We study the intersection theory on geometric invariant theory quotients for which semistability is not necessarily the same as stability. We give formulas for the pairing of intersection cohomology classes and for the intersection numbers on the partial desingularization of a singular quotient. (Joint work with L. Jeffrey, F. Kirwan and J. Woolf).

TARA HOLM
Massachusetts Institute of Technology

The mod 2 equivariant cohomology of the real locus of a Hamiltonian T-Space

Suppose M is a Hamiltonian T^n-space, and that σ is an anti-symplectic involution compatible with the T action. The real locus of M is X, the fixed point set of σ. The standard example is when M is a complex manifold, σ is a complex conjugation, and X is the honest real part of M. Duistermaat uses Morse theory to give a nice description of the ordinary cohomology of X in terms of the cohomology of M. There is a residual $T_2^n = (Z/2Z)^n$ action on X, and we can use Duistermaat's result, as well as some general facts about equivariant cohomology, to prove an equivariant analogue of Duistermaat's theorem. In some cases, we can also extend theorems of Goresky-Kottwitz-MacPherson and Goldin-Holm to the real locus. (This is joint work with Daniel Biss and Victor Guillemin)

DAVID METZLER
University of Florida

Orbifold K-theory

We compare several different definitions of (equivariant) K-theory for orbifolds and examine how far one can get with the naive geometric definition. We note how M. Vergne's index formula fits into the K-theoretic picture.

JOHN MILLSON
University of Maryland

Eigenvectors of sums, singular values and invariant factors of products and spaces of nonpositive curvature

In this talk I will discuss joint work with Bernhard Leeb and Misha Kapovich. I will explain why the three linear algebra problems discussed by Fulton in his Bulletin article (BAMS **37**) all lead to the same set of inequalities - we do not discuss the fourth problem of decomposing tensor products. I will then discuss the possible generalizations to other semisimple algebraic groups G split over $\mathbb{Q}$. At the moment we have only partial results for the p-adic case for general G. This is the generalization of the problem of specifying the invariant factors of a product in advance. In terms of geometry it involves constructing n-gon linkages with given "side-lengths" and *given vertex type* in Bruhat-Tits buildings.

LEONOR GODINHO
Instituto Superior Tecnico

Equivariant cohomology and Hamiltonian circle actions

The problem of determining which symplectic circle actions are Hamiltonian is still open. An obvious necessary condition on a compact manifold is that the action must have fixed points corresponding to the critical points of the Hamiltonian function. McDuff showed that this condition was not in general sufficient by constructing a symplectic six-manifold with a circle action which had fixed points but was not Hamiltonian. However, there are no known examples of symplectic non-Hamiltonian circle actions with isolated fixed points (the fixed point sets in McDuff's example are tori). It is conceivable then that an action with isolated fixed points is Hamiltonian. This was recently proved by Tolman and Weitsman for semi-free circle actions. We will discuss their methods which use integration in equivariant cohomology and explain how they can be extended to some non-semifree cases.

SEBASTIEN RACANIERE
IRMA de Strasbourg

Equivariant cohomology of $SU(n)^{2g}$ and Kirwan's map

Let X be a holomorphic curve of genus g and $\mathfrak{m}$ the moduli space of rank n degree k stable holomorphic bundles on X with fixed determinant, $n \geq 2$, $(n, k) = 1$. A generalisation of the construction proposed by M. Mumford and P.Newstead, M. S. Narasimhan and C. S. Seshadri when $n = 2$, $k = 1$, of a universal bundle on $X \times \mathfrak{m}$ is given. The space $\mathfrak{m}$ can be seen as a quasi-hamiltonian reduction of the quasi-hamiltonian space $SU(n)^{2g}$. Using the previous construction of the universal bundle, some results are obtained about the image of the equivalent of the Kirwan map for quasi-hamiltonian spaces, the strongest results being when $n = 2$.

ALLEN KNUTSON
University of California, Berkeley

Non-differentiable action variables can come from algebraic families

The "Thimm trick" of symplectic geometry extends a Hamiltonian K-action on a symplectic manifold to a $K \times T$-action – except at some bad points, where the action is not well-defined, but "its moment map" still is. (The map isn't differentiable, just continuous, so doesn't define a Hamiltonian vector field.)

The algebraic geometers have an analogous construction called the "Vinberg asymptotic cone." I will show (1) that there in the complex algebraic case, there is a natural continuous map from the original space to its asymptotic cone and (2) in a certain limit the Thimm moment map is the composite of this with the honest moment map on the asymptotic cone. Even when these maps are smooth, they can be interesting, in showing distinct projective varieties are symplectomorphic.