Quantum gates in Kane's model based on adiabatic controlling processes and verification of its physical realization

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Motivation & Aim

- **Physical realization of the Kane's model (Nature 393, 133 (1998))**
  - **difficult, but challenging problem !!**
  - **possibility: Single Ion Injection Method (SII)**

    (T. Shinada, *et al.*, **Jpn. Appl. Phys. 41, L287 (2002)**

    (Waseda University))
- **Control of quantum systems by classical manipulation**
  - **Kane's model: change the gate voltage**

    ⇒ change the strength of interaction (locally)

We discuss the construction of quantum gates,

based on rigorous analyses in the proposed model.

Model

N–qubit system

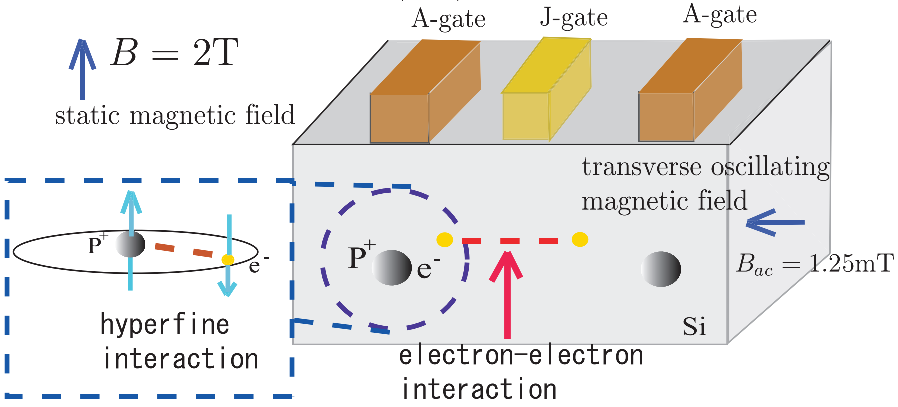
$$H = H_0 + H_{ac},\ H_0 = \sum_{i=1}^{N} H^i + \sum_{\langle i,j \rangle} J_{ij} \boldsymbol{\sigma}^{ie} \cdot \boldsymbol{\sigma}^{je},\ H_{ac} = \sum_{i=1}^{N} H_{ac}^i$$

$$H^i = \mu_B B \sigma_z^{ie} - g_n \mu_n B \sigma_z^{in} + A_i \boldsymbol{\sigma}^{ie} \cdot \boldsymbol{\sigma}^{in},\ H_{ac}^i = B_{ac} \boldsymbol{m}^i \cdot (\mu_B \boldsymbol{\sigma}^{ie} - g_n \mu_n \boldsymbol{\sigma}^{in})$$

- A_i: hyperfine interaction (HF) (on the i–th dopant)
- J_{ij}: electron–electron exchange interaction (EE) (between i–th and j–th dopant), $j = i+1$
- (gate voltage)$= 0 \Rightarrow A_i = A_0,\ J_{ij} = 0$
  ($2A_0/h = 58$ MHz)
- $\mu_B B = 1.158\,\text{meV}$,
  $g_n \mu_n B = 7.135 \times 10^{-5}\,\text{meV}$
- $\boldsymbol{m}^i = (\cos(\omega_{ac} t), -\sin(\omega_{ac} t), 0)$

B. E. Kane, Nature 393, 133 (1998)

$T \simeq 100\,\text{mK}$



Method – adiabatic ctrolling processes –

- A_i, J_{ij}: **adiabatic controlling processes**
- **transverse magnetic field: instantaneously swich on–off** ($\because$ B_{ac}/B: **small**)

$$2A_0/\mu_B B \simeq 2.07 \times 10^{-4},\ g_n \mu_n B/\mu_B B \simeq 0.62 \times 10^{-4},\ J \sim \mu_B B,\ B_{ac}/B \simeq 10^{-3}$$

⋆ **phase shift for** i**–th qubit**

⋆ **spin flip for** i**–th qubit**

⋆ **controlled-Z between** i**–th and** j**–th qubits** $(j = i + 1)$

Diagonalization of Hamiltonian

- H^i: **related with the dynamics for one qubit**

  $S^i = (\sigma_z^{ie} + \sigma_z^{in})/2$, $[S^i, H^i] = 0$

  $$H^i = E^i_{\uparrow 0}|u^i_{\uparrow 0}\rangle\langle u^i_{\uparrow 0}| + E^i_{\uparrow 1}|u^i_{\uparrow 1}\rangle\langle u^i_{\uparrow 1}| + E^i_{\downarrow 0}|u^i_{\downarrow 0}\rangle\langle u^i_{\downarrow 0}| + E^i_{\downarrow 1}|u^i_{\downarrow 1}\rangle\langle u^i_{\downarrow 1}|$$

  $|u^i_{\downarrow 0}\rangle = (-2A_i|\uparrow 1\rangle + (\epsilon + \sqrt{\epsilon^2 + 4A_i^2})|\downarrow 0\rangle)/N_i$, $|u^i_{\downarrow 1}\rangle = |\downarrow 1\rangle$

  $E^i_{\uparrow 0} = \epsilon - 2g_n\mu_n B + A_i$, $E^i_{\uparrow 1} = -A_i + \sqrt{\epsilon^2 + 4A_i^2}$,

  $E^i_{\downarrow 0} = -A_i - \sqrt{\epsilon^2 + 4A_i^2}$, $E^i_{\downarrow 1} = -\epsilon + 2g_n\mu_n B + A_i$ $(\epsilon = \mu_B B + g_n\mu_n B)$

- $H^{ij} = H^i + H^j + J_{ij}\boldsymbol{\sigma}^{ie} \cdot \boldsymbol{\sigma}^{je}$, $j = i + 1$

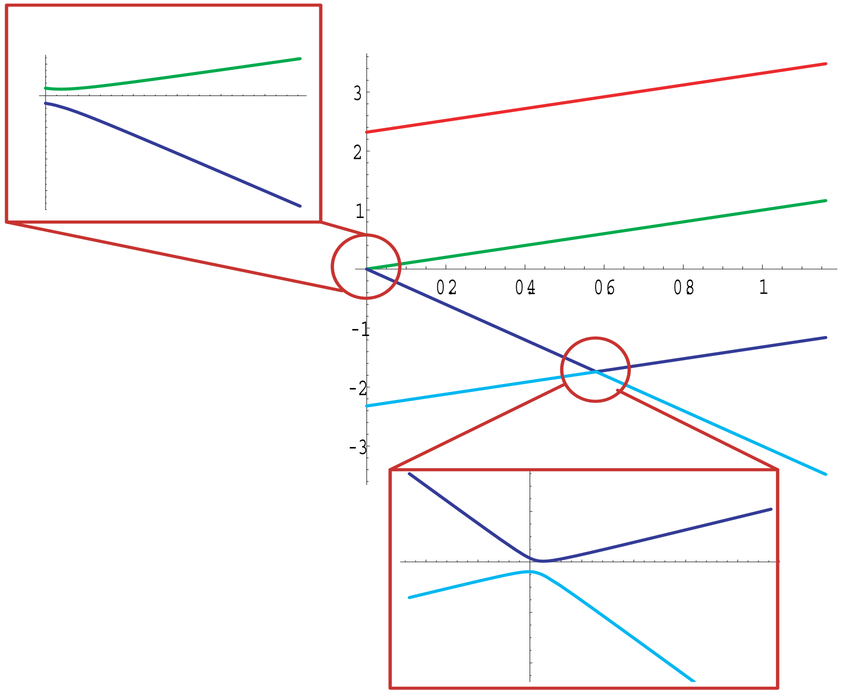
  $S^{ij} = S^i + S^j$, $[S^{ij}, H^{ij}] = 0$, P^{ij}: **exchange of labels for identical particles**

$$\boxed{A_i = A_j \iff [P^{ij}, H^{ij}] = 0}$$

$\Rightarrow H^{ij}$: **block diagonal form** $\Rightarrow$ **analyticaly diagonalization**

quantum number: $S^{ij} \rightarrow s = 2, 1, 0, -1, -2$, $P^{ij} \rightarrow p = +, -$

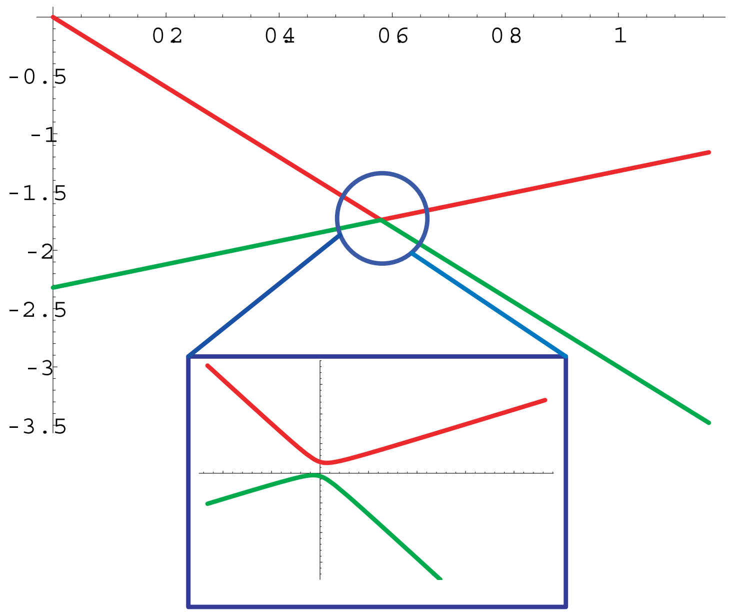
Energy level I



vertical axis: eigenvalues [meV] for $(s, p) = (0, +)$ **, horizonal axis:** J **[meV],**

cyan line: $|u^{i}_{\downarrow 0}\rangle|u^{i+1}_{\downarrow 0}\rangle$ **(** $J = 0$ **)**

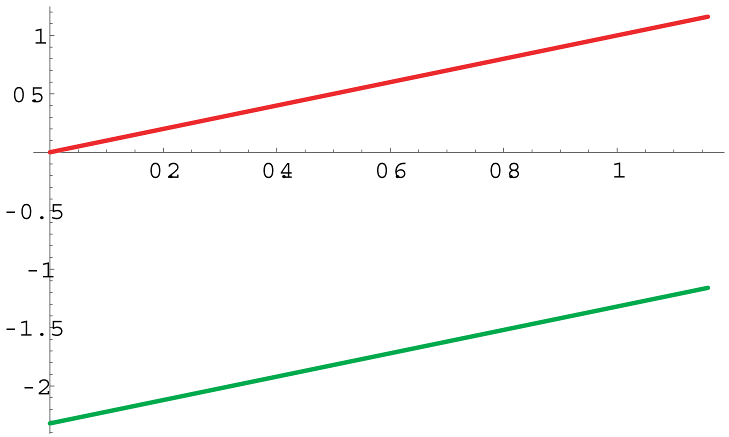
Energy level II

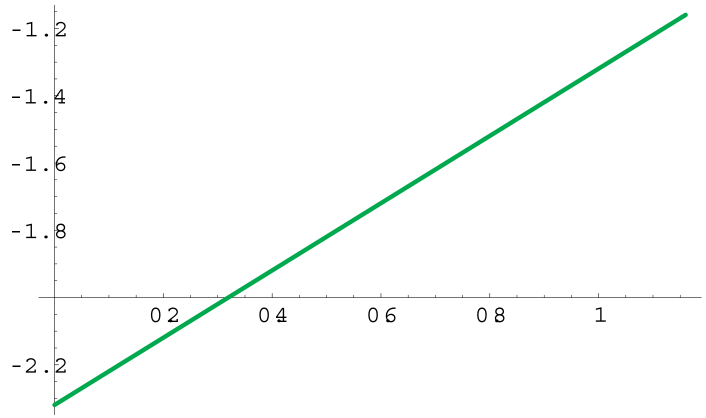


vertical axis: eigenvalues [meV] for $(s, p) = (-1, -)$ **, horizonal axis** J **[meV],**

green line: $(|u^i_{\downarrow 0}\rangle |u^{i+1}_{\downarrow 1}\rangle - |u^i_{\downarrow 1}\rangle |u^{i+1}_{\downarrow 0}\rangle)/\sqrt{2}(J = 0)$

Energy level III





- **left figure, vertical axis: eigenvalues [meV] for** $(s,p) = (-1,+)$**, horizonal axis:** J **[meV],**

  green line: $(|u^i_{\downarrow 0}\rangle|u^{i+1}_{\downarrow 1}\rangle + |u^i_{\downarrow 1}\rangle|u^{i+1}_{\downarrow 0}\rangle)/\sqrt{2}$ **(**$J = 0$**)**
- **right figure, vertical axis: eigenvalue [meV] for** $(s,p) = (-2,+)$**, horizonal axis:** J **[meV],**

  green line: $|u^i_{\downarrow 1}\rangle|u^{i+1}_{\downarrow 1}\rangle$ **(**$J = 0$**)**

Representation of Quantum Information

Eigenvector for H^i: $|u^i_{\uparrow 0}\rangle (= |\downarrow 0\rangle)$, $|u^i_{\uparrow 1}\rangle (\simeq |\uparrow 1\rangle)$, $|u^i_{\downarrow 0}\rangle (\simeq |\downarrow 0\rangle)$, $|u^i_{\downarrow 1}\rangle (= |\downarrow 1\rangle)$

- **Initialization:** $T \simeq 100\,\mathrm{mK} \Rightarrow \rho = \dfrac{1}{Z} \exp\left(-\beta \sum_{i=1}^{N} H^i \right) \simeq \bigotimes_{i=1}^{N} |u_{\downarrow 0}\rangle_i \langle u_{\downarrow 0}|$
- $|E^i_{\downarrow 0} - E^i_{\downarrow 1}|$**: characterized by** A_0

  $\Rightarrow$ **the gate construction through the control of HF**

$$|u^i_{\downarrow 0}\rangle \doteq |0\rangle_L, \;\; |u^i_{\downarrow 1}\rangle \doteq |1\rangle_L$$

- **controlled operation (controlled–Z)** $(j = 1 + 1)$

  computational basis: $|u^i_{\downarrow 0}\rangle |u^j_{\downarrow 0}\rangle$, $|u^i_{\downarrow 0}\rangle |u^j_{\downarrow 1}\rangle$, $|u^i_{\downarrow 1}\rangle |u^j_{\downarrow 0}\rangle$, $|u^i_{\downarrow 1}\rangle |u^j_{\downarrow 1}\rangle$

$$|v_1\rangle \equiv |u^i_{\downarrow 0}\rangle |u^j_{\downarrow 0}\rangle \leftrightarrow (s,p) = (0,+), \;\; |v_4\rangle \equiv |u^i_{\downarrow 1}\rangle |u^j_{\downarrow 1}\rangle \leftrightarrow (s,p) = (-2,+),$$

$$|v_\pm\rangle \equiv (|u^i_{\downarrow 0}\rangle |u^j_{\downarrow 1}\rangle \pm |u^i_{\downarrow 1}\rangle |u^j_{\downarrow 0}\rangle)/\sqrt{2} \leftrightarrow (s,p) = (-1,\pm)$$

Each vector belongs to the different subspace !

Result 1 – phase shift –

$J_{kl}=0$, $A_i(t)=A_0(1-a\sin(\pi t/T_{op}))$, $A_k=A_0$ $(k\neq i)$, T_{op}: **operation time**, a: **parameter**

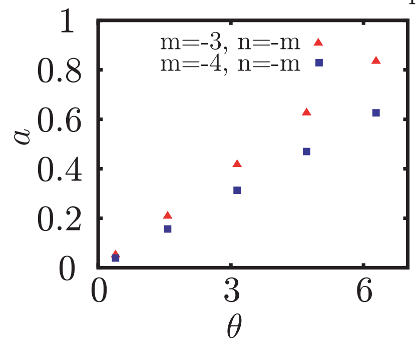
- **phase difference between** $|u^i_{\downarrow 0}\rangle$(**adiabatic**) **and** $|u^i_{\downarrow 1}\rangle$(**eigenstate**)

i**th qubit:** $\Theta_i=\frac{T_{op}}{\hbar}\left(-2\int_0^1 A_i(\tau)d\tau-\int_0^1\sqrt{\epsilon^2+4A_i(\tau)^2}d\tau+\epsilon-2g_n\mu_nB\right)$

others: $\Theta_0=\frac{T_{op}}{\hbar}\left(-2A_0-\sqrt{\epsilon^2+4A_0^2}+\epsilon-2g_n\mu_n\right)$

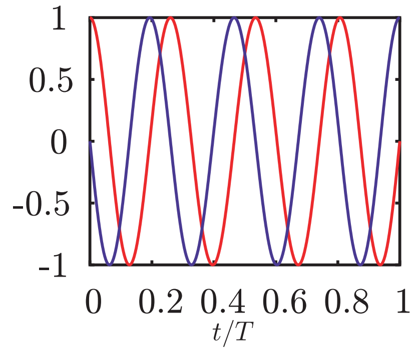
$\Rightarrow\theta=\Theta_i-\Theta_0-2n\pi$, $(\Theta_0=2m\pi,\ m\in\mathbb{Z},\ n\in\mathbb{Z})\rightarrow$ **determine the value of** a

numerical solution of the equation of a for given (m,n)



$A_i\searrow 0.6A_0$ at most $(m=-4,\ n=-m)$

$\Rightarrow$ **any phase shift gate**



$\theta=\pi/2\ \Rightarrow$

$a=0.157$ $(m=-n=-4)$

$T=0.0438\mu s$

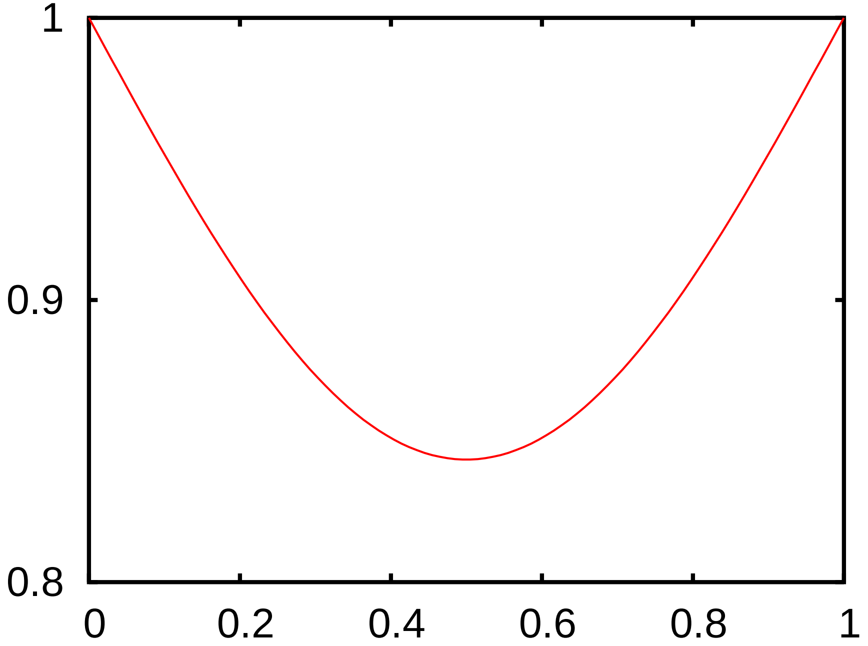
$\cos\theta$

$\sin\theta$

$|u_{\downarrow 1}\rangle$: **eigenstate**, $|\langle u_{\downarrow 0}|\psi(T_{op})\rangle|=1+C/\epsilon T_{op}$, $|\langle u_{\uparrow 1}|\psi(T_{op})\rangle|=C/\epsilon T_{op}$, $C\simeq 10^{-5}$,

$\epsilon T_{op}\simeq 10^3\Rightarrow$ **adiabatic approximation: good**$\Rightarrow$ **error** $\sim 10^{-8}$

profile of A_i



vertical axis: A_i/A_0, **horizonal axis:** t/T_{op}

Result 2 – spin sfip –

Larmor resonance frequency

$$\hbar\omega_{ac} = -\epsilon + 2g_n\mu_n B + 2A + \sqrt{\epsilon^2 + 4A^2}$$

$A_i = A$, $A_j = A_0 (j \neq i)$: **control the resonance condition locally**

operation time $T_{op} = \hbar\alpha/\nu_\theta B_{ac}$, $\nu_\theta \simeq g_n\mu_n$, α: **angle (i.e.,** $e^{-i\alpha\sigma_x}$**)**

This method correspond to Hill & Goan's work **PRA 012321 (2003)**

⋆ **Hamiltonian in the rotating frame**

$\Rightarrow$ $^{\exists}$ **the term related with transition between the different electron spin states**

error: $\|\psi_{rot}(t) - \psi_d(t)\| \leq \left(2 + \frac{\mu^i_{-\theta}\alpha}{\nu^i_\theta}\right) \frac{2\mu^i_{-\theta} B_{ac}}{\epsilon - g_n\mu_n B_{ac}\cos\theta^i}$

$\mu^i_\theta = \mu_B \cos\theta^i - g_n\mu_n \sin\theta^i$, $\nu^i_\theta = \mu_B \sin\theta^i + g_n\mu_n \cos\theta^i$ $\cos\theta^i = (\epsilon + \sqrt{\epsilon^2 + 4A_i^2})/N_i$, $\sin\theta^i = 2A_i/N_i$

$B_{ac}/B \sim 10^{-3}$: **often used** $\Rightarrow$ **error** $\rightarrow$ **large !!**

Result 3 – controlled-Z –

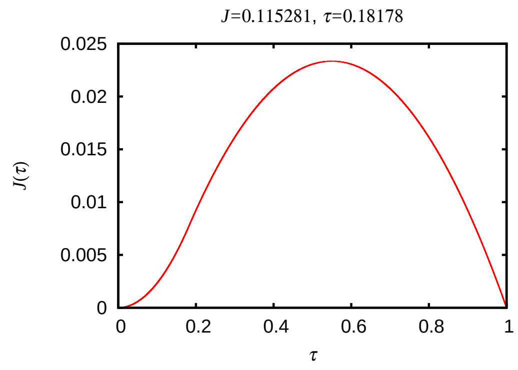
- For example

$$J(\tau) = \begin{cases} J_c \alpha \tau^2 & 0 \le \tau < \tau_c \\ J_c\{\beta - (\tau - \tau_0)^2\} & \tau_c < \tau \le 1 \end{cases}$$

$$\tau = t/T_{op},\ \alpha = (1-\tau_c)^2/\{\tau_c(1-\tau_c)\},$$

$$\beta = (1-\tau_c)^2/(2-\tau_c)^2,\ \tau_0 = 1/(2-\tau_c)$$

parameter: τ_c, J_c, and T_{op}



adiabatic time evolution $\Leftarrow$ **analytical calculation !**

$$|v_1\rangle \to e^{i\delta_1}|v_1\rangle,\ |v_\pm\rangle \to e^{i\delta_\pm}|v_\pm\rangle = e^{i\delta_1}e^{i(\delta_\pm-\delta_1)}|v_\pm\rangle,\ |v_4\rangle \to e^{i\delta_4}|v_4\rangle = e^{i\delta_1}e^{i(\delta_4-\delta_1)}|v_4\rangle$$

controlled–Z $\iff \delta_\pm - \delta_1 = 2m_{\pm,1}\pi,\ \delta_4 - \delta_1 = 2m_{4,1}\pi + \pi,\ m_{\pm,1}, m_{4,1} \in \mathbb{Z}$

J_c/ϵ	τ_c	$m_{4,1}$	$m_{+,1}$	$m_{-,1}$	T_{op} [μs]
0.115281	0.181788	25	25	−24	0.0054
0.695156	0.0575511	50	50	−49	0.0054

adiabatic approximation: good (numerically)

Matrix for the controlled-Z

$$e^{i\delta_1}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a_+ & a_- & 0 \\ 0 & a_- & a_+ & 0 \\ 0 & 0 & 0 & a \end{pmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

$$a_+ = (e^{i(\delta_+ - \delta_1)} + e^{i(\delta_- - \delta_1)})/2 = (e^{i2m_{+,1}\pi} + e^{i2m_{-,1}\pi})/2 = 1$$

$$a_- = (e^{i(\delta_+ - \delta_1)} - e^{i(\delta_- - \delta_1)})/2 = (e^{i2m_{+,1}\pi} - e^{i2m_{-,1}\pi})/2 = 0$$

$$a = e^{i(\delta_4 - \delta_1)} = e^{i2m_{4,1}\pi} e^{i\pi} = -1$$

$$\Downarrow$$

$$e^{i\delta_1}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

Conclusion

- **We discuss the construction of the quantum gates through the adiabatic controlling processes**
  - **phase shift gate: we determine the good parameter.**
    adiabatic approximation: good $\rightarrow$ very small error
  - **spin flip gate: standard value ($B_{ac}/B \sim 10^{-3}$) $\Rightarrow$ very large error**
    improvement: the value of B_{ac}/B $\Rightarrow$ smaller
    $\Rightarrow$ But, the operation time increases $\Rightarrow$ $^{\exists}$Optimal value ?
  - **controlled-Z gate: we show a possible several sets of parameters.**
    Examine much more.

perspective

- **The theoretical estimation of the required accuracy of "ion injection" which enables us to perform the quantum computation.**