Geometric Quantum Operations in Two-Level Atomic Systems

M. Tian, Z. W. Barber, J. A. Fischer, and Wm. R. Babbitt

Physics Department, Montana State University—Bozeman, MT 59717,
Tian@physics.montana.edu

Quantum Information and Quantum Control, July 19~23, 2004, Toronto

Introduction

The geometric manipulation of the quantum states of qubits has the potential to produce a full set of robust universal quantum operations for quantum computing. We have investigated the manipulation of the quantum states of two-level rare-earth atoms using laser-controlled geometric phases. A set of universal single qubit operations has been designed using resonant laser pulses. An operation equivalent to a phase gate has been demonstrated with thulium ions doped in yttrium aluminum garnet crystal.

Wave function & Bloch representation

2-level atom

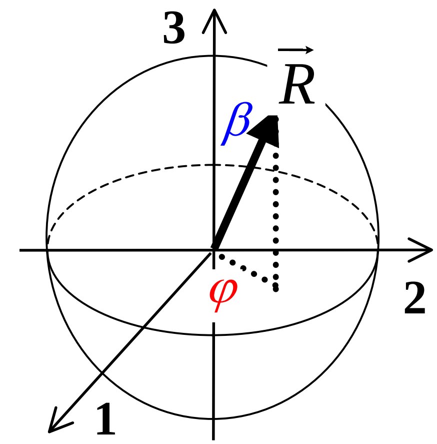
$$|1\rangle$$

$$|0\rangle$$

Wave function

$$|\psi\rangle = e^{i\gamma}\begin{pmatrix} \cos\frac{\beta}{2} \\ e^{-i\varphi}\sin\frac{\beta}{2} \end{pmatrix}$$

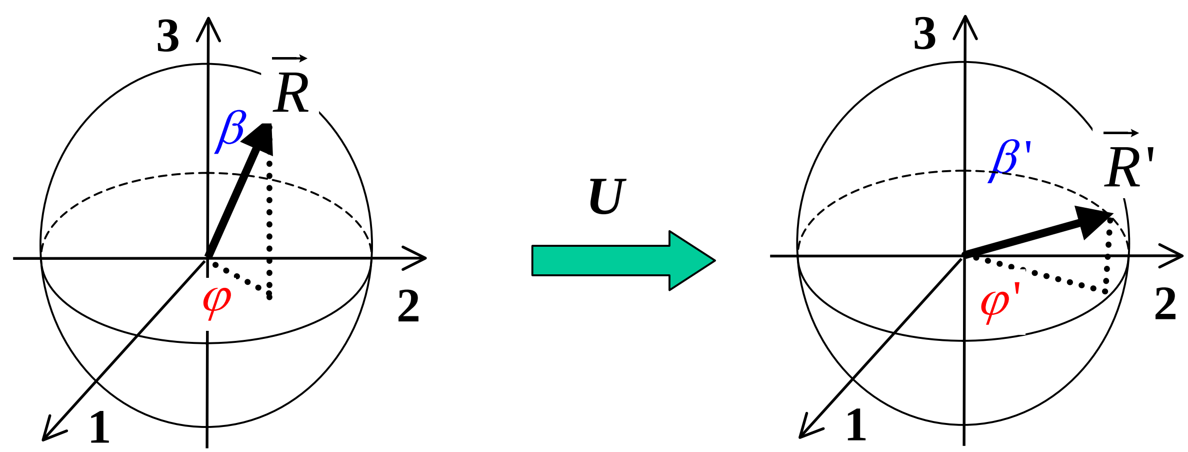
Bloch representation



Single qubit operation *U* alters Bloch vector's orientation

$$|\psi'\rangle = U|\psi\rangle$$

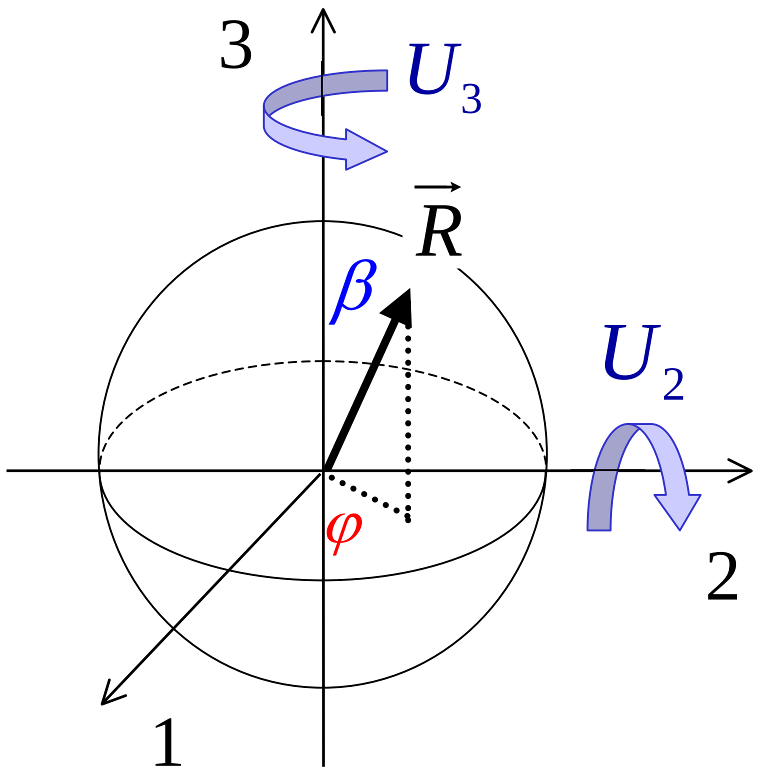
$$\vec{R}' = U\vec{R}$$



Universal single qubit operation

Any arbitrary single qubit operation

made of two basic rotations: U_2 and U_3



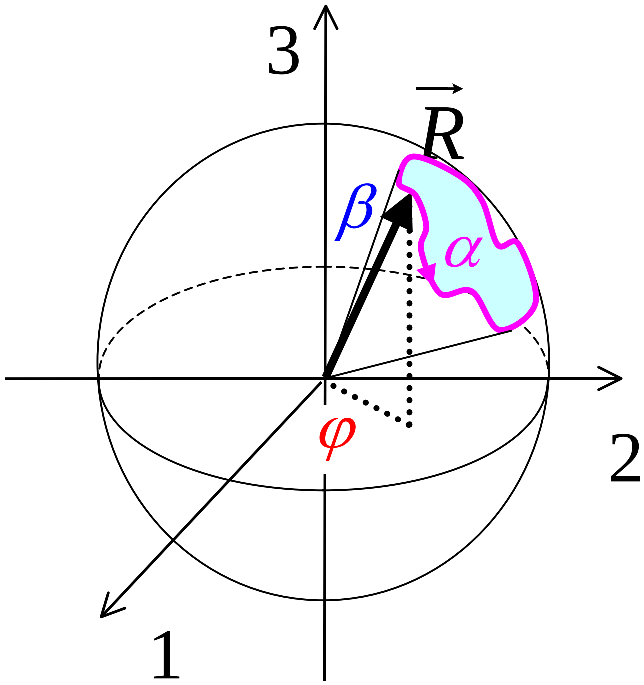
$$U = U_3(\delta_3)U_2(\delta_2)U_3(\delta_1)$$

$$U_3(\delta) = \begin{pmatrix} e^{i\delta/2} & 1 \\ 1 & e^{-i\delta/2} \end{pmatrix},$$

$$U_2(\delta) = \begin{pmatrix} \cos(\delta/2) & \sin(\delta/2) \\ -\sin(\delta/2) & \cos(\delta/2) \end{pmatrix}$$

The basic Bloch rotations can be accomplished through controlled geometric phase manipulation.

Geometric phase



After a cyclic evolution, the wave function gains geometric phase

$$\Delta\gamma = -\alpha / 2$$

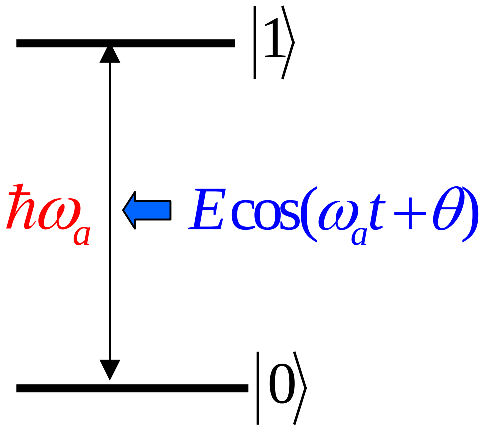
Geometric property:

- **determined solely by the amount of the solid angle enclosed by the evolution path,**
- **independent of driving Hamiltonian, the quantum state of the system, and the shape of the path.**

M.V. Berry, Proc. R. Soc, Ser A 392, 45 (1984), Y. Aharonov and J. Anandan . Phys. Rev. Lett. 58, 1593 (1987)

Geometric Phase in three-level atomic system, Phys. Rev. A 67. 011403 (R) (2003)
Geometric manipulation of the quantum states of two-level atoms, Phys. Rev. A 69. 050301 (R) (2004)

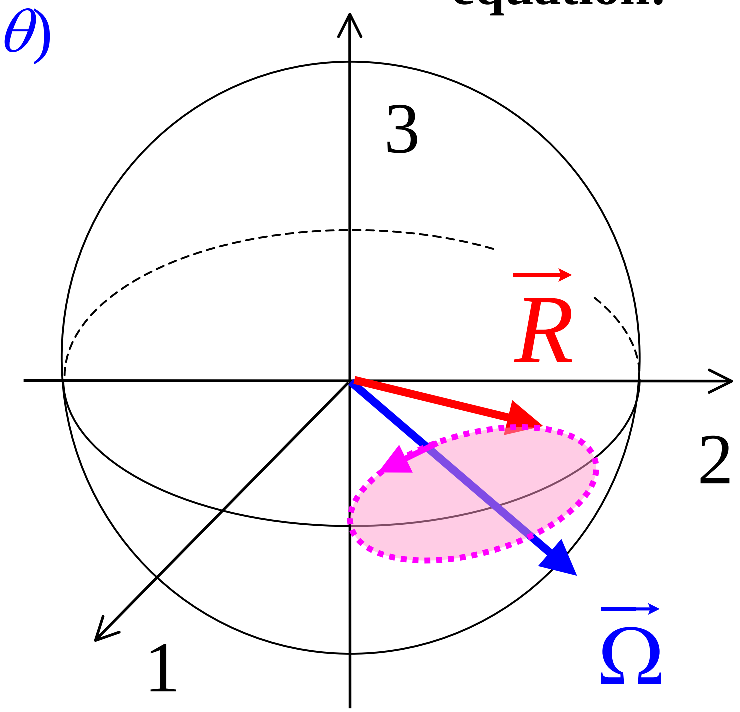
Two-level atom driven by laser pulse



Two-level atom

Optical Bloch equation:

$$\frac{d\vec{R}}{dt} = \vec{\Omega}\times\vec{R}$$



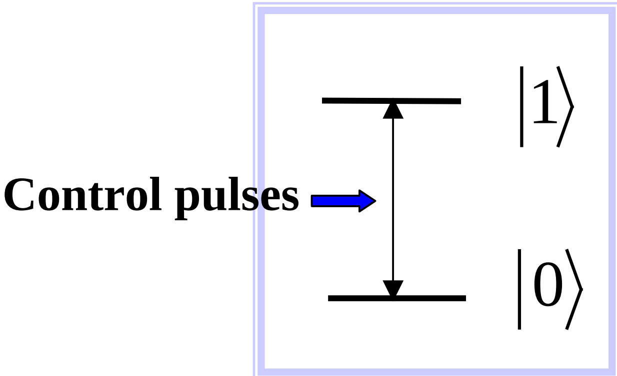
$$\vec{\Omega}:[-\Omega\cos\theta,\ -\Omega\sin\theta,\ 0]$$

Rabi frequency: $\Omega=\mu E/\hbar$,

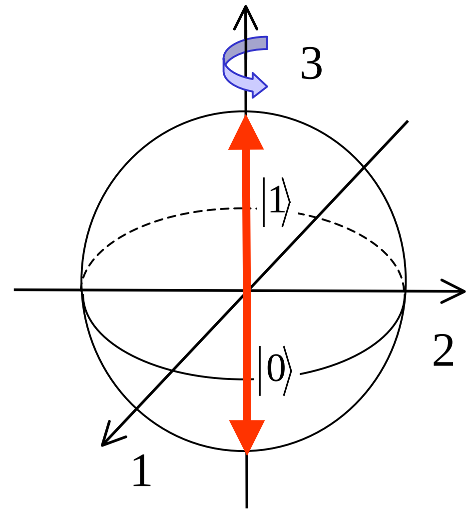
Pulse Area: $A=\Omega\tau_p$.

Two basic rotations of an arbitrary Bloch vector for an arbitrary angle can be accomplished through pure geometric phase changes driven by laser pulse sequence satisfying $\vec{\Omega}\perp\vec{R}$ **along the evolution path.**

Basic rotation $U_3(\delta)$



$$U_3(\delta) = \begin{pmatrix} e^{i\delta/2} & 1 \\ 1 & e^{-i\delta/2} \end{pmatrix}$$



$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U_3(\delta)\begin{pmatrix} \cos\frac{\beta}{2} \\ e^{-i\varphi}\sin\frac{\beta}{2} \end{pmatrix} = \begin{pmatrix} e^{i\delta/2}\cos\frac{\beta}{2} \\ e^{-i\delta/2}e^{-i\varphi}\sin\frac{\beta}{2} \end{pmatrix}$$

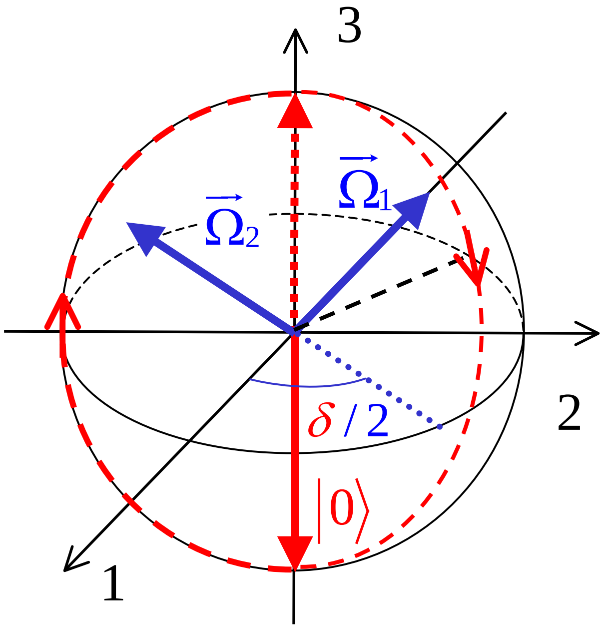
$$U_3(\delta)|0\rangle = e^{i\delta/2}|0\rangle \qquad U_3(\delta)|1\rangle = e^{-i\delta/2}|1\rangle$$

Control pulse sequence (2 pulses): π pulse ($\theta=0$), π pulse with $\theta = \delta/2$

$U_3(\delta)$ Operation on $|0\rangle$ and $|1\rangle$

$\vec{\Omega}_1$: π pulse (θ=0)

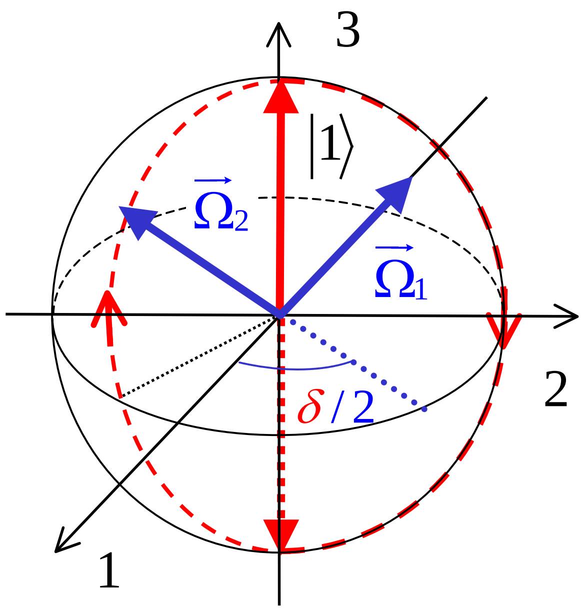
$\vec{\Omega}_2$: π pulse ($\theta=\delta/2$)



Solid angle: 2π-δ

Geometric phase: $-\pi+\delta/2$

$$|0\rangle \Rightarrow e^{-i\pi+i\delta/2}|0\rangle$$



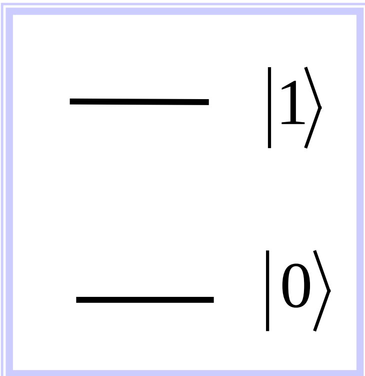
Solid angle: $2\pi+\delta$

Geometric phase: $-\pi-\delta/2$

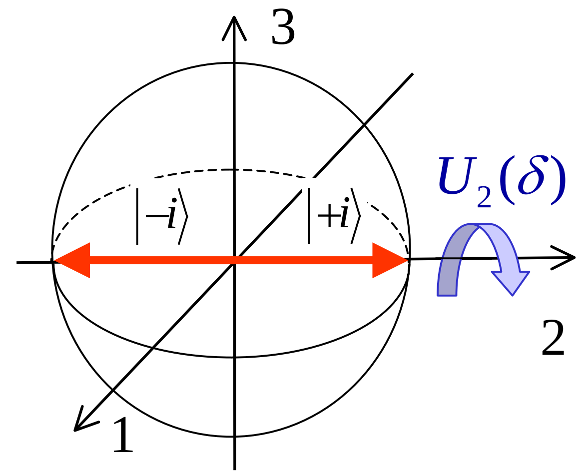
$$|1\rangle \Rightarrow e^{-i\pi-i\delta/2}|1\rangle$$

$$U_3(\delta)|0\rangle = e^{i\delta/2}|0\rangle \qquad U_3(\delta)|1\rangle = e^{-i\delta/2}|1\rangle$$

Basic rotation: $U_2(\delta)$



$$U_2(\delta) = \begin{pmatrix} \cos(\delta/2) & \sin(\delta/2) \\ -\sin(\delta/2) & \cos(\delta/2) \end{pmatrix}$$



$$U_2(\delta)\left|+i\right\rangle = e^{-i\delta/2}\left|+i\right\rangle$$

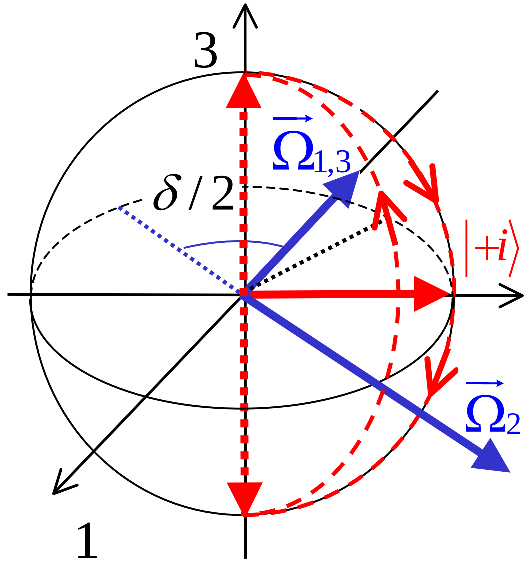
$$U_2(\delta)\left|-i\right\rangle = e^{i\delta/2}\left|-i\right\rangle$$

$$\left|+i\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, \quad \left|-i\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Three-pulse control sequence

$\pi/2$ pulse (θ=0), π pulse ($\theta=\pi+\delta/2$) , and $\pi/2$ pulse (θ=0)

Control pulse sequence for $U_2(\delta)$



$\vec{\Omega}_1$: $\pi/2$ pulse (θ=0)

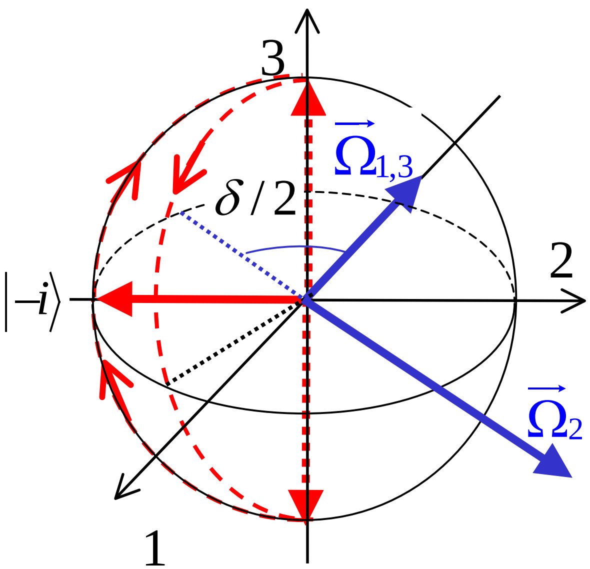
$\vec{\Omega}_2$: π pulse ($\theta=\pi+\delta/2$)

$\vec{\Omega}_3$: $\pi/2$ pulse (θ=0)

Solid angle: δ,

Geophase: -δ/2

$$|+i\rangle \Rightarrow e^{-i\delta/2}|+i\rangle$$



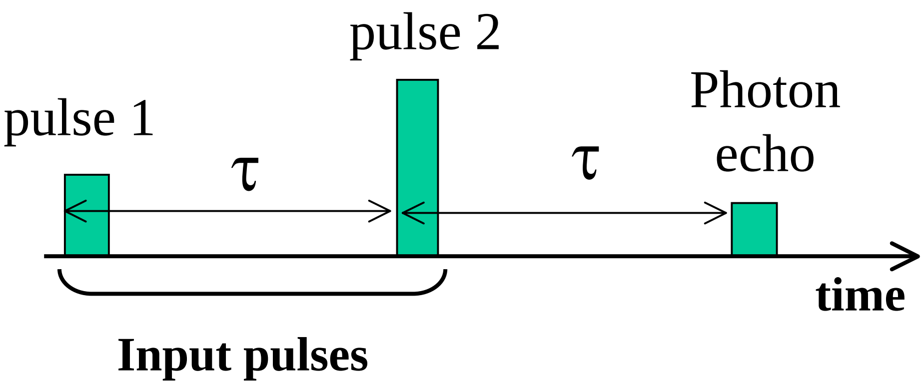
Solid angle : 4π-δ,

Geophase: δ/2

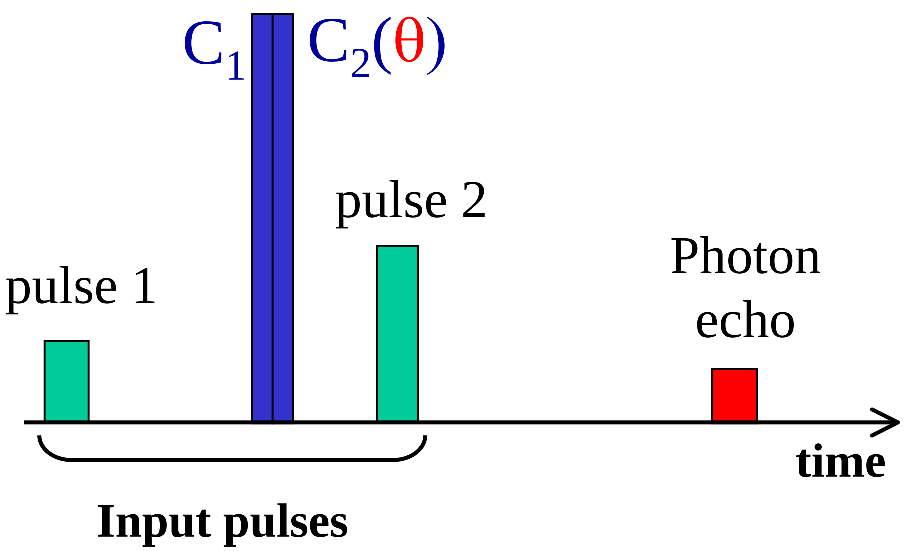
$$|-i\rangle \Rightarrow e^{i\delta/2}|-i\rangle$$

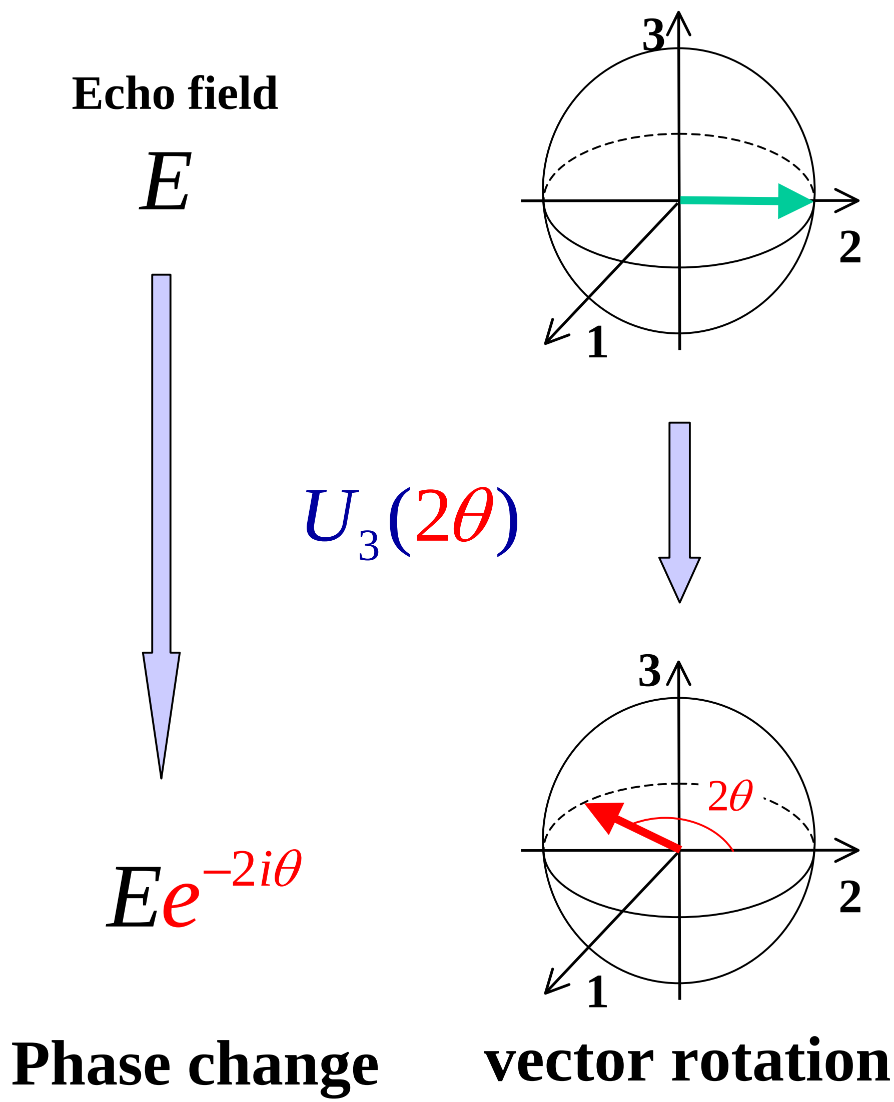
Observation of the rotation U_3

Photon echo process

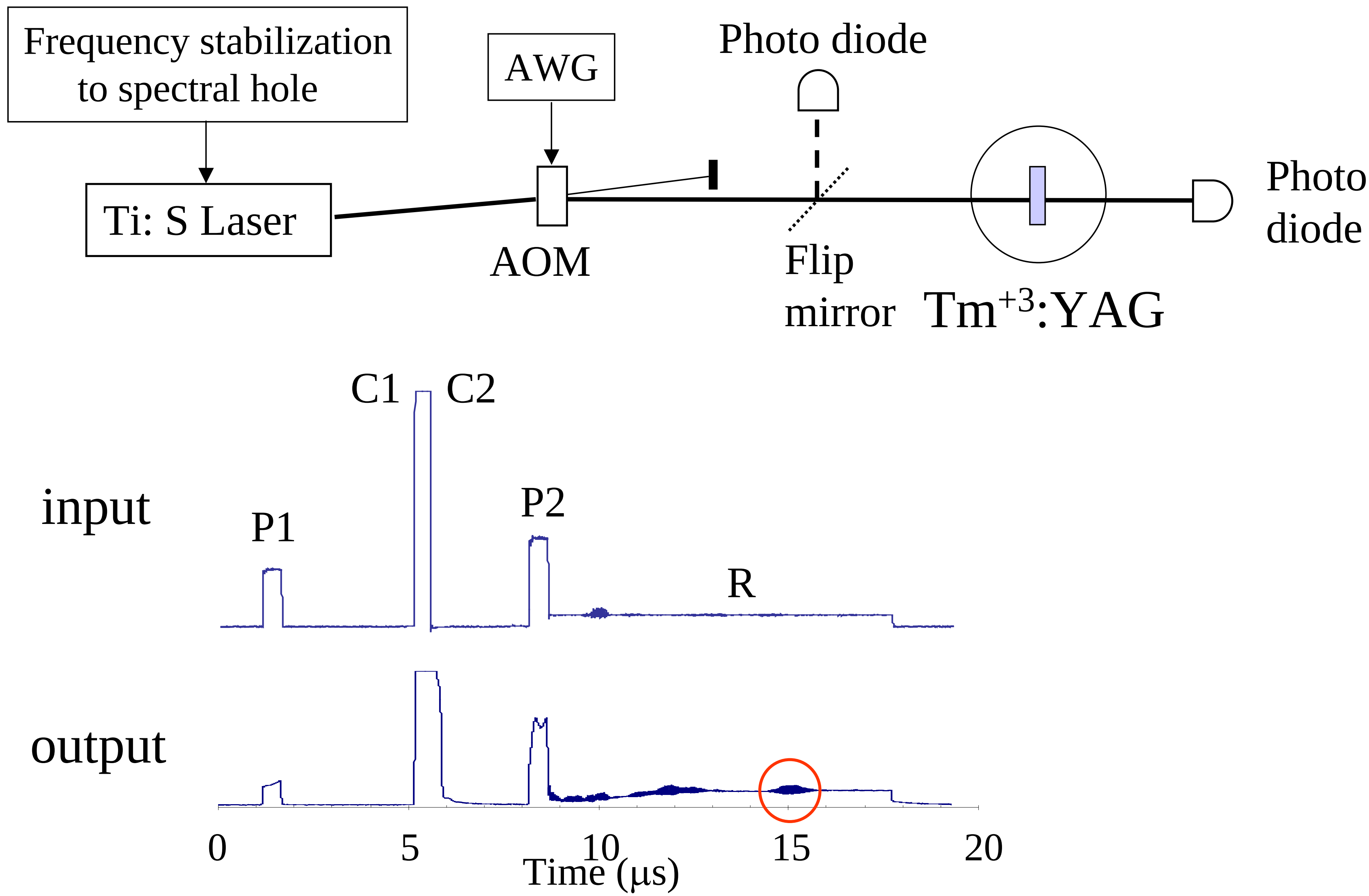


Add 2 π control pulses

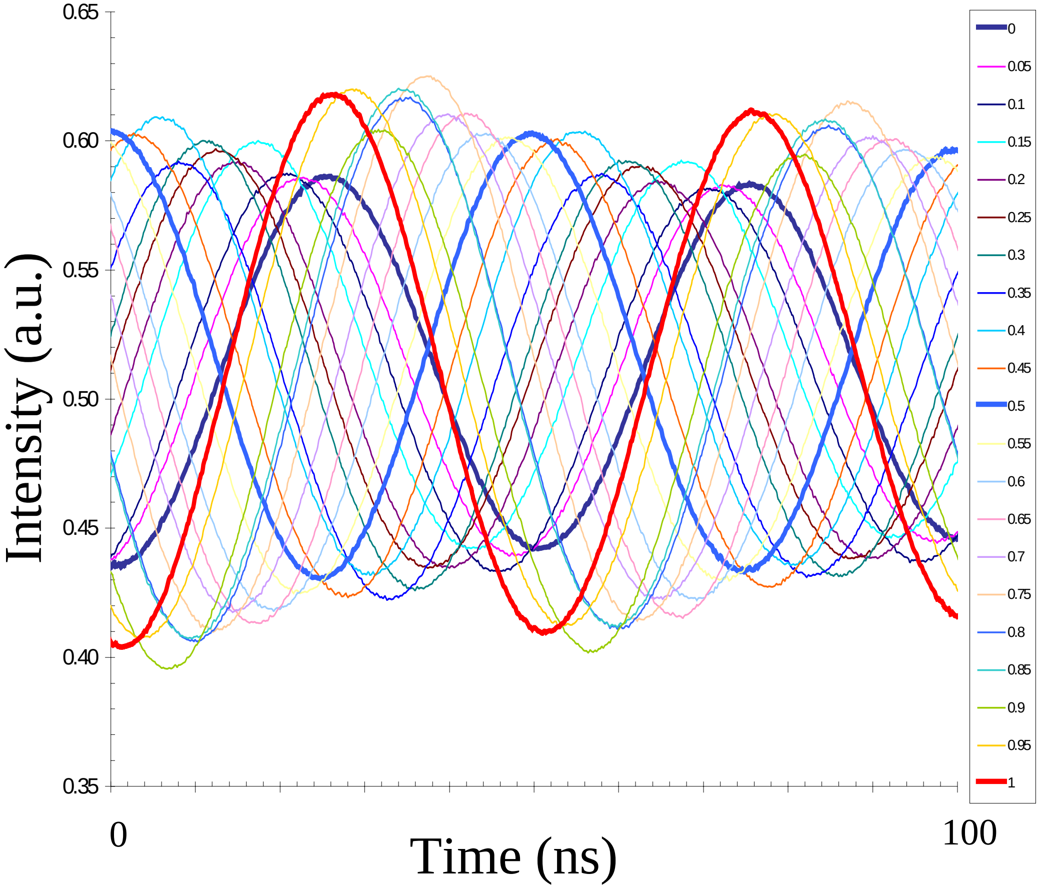




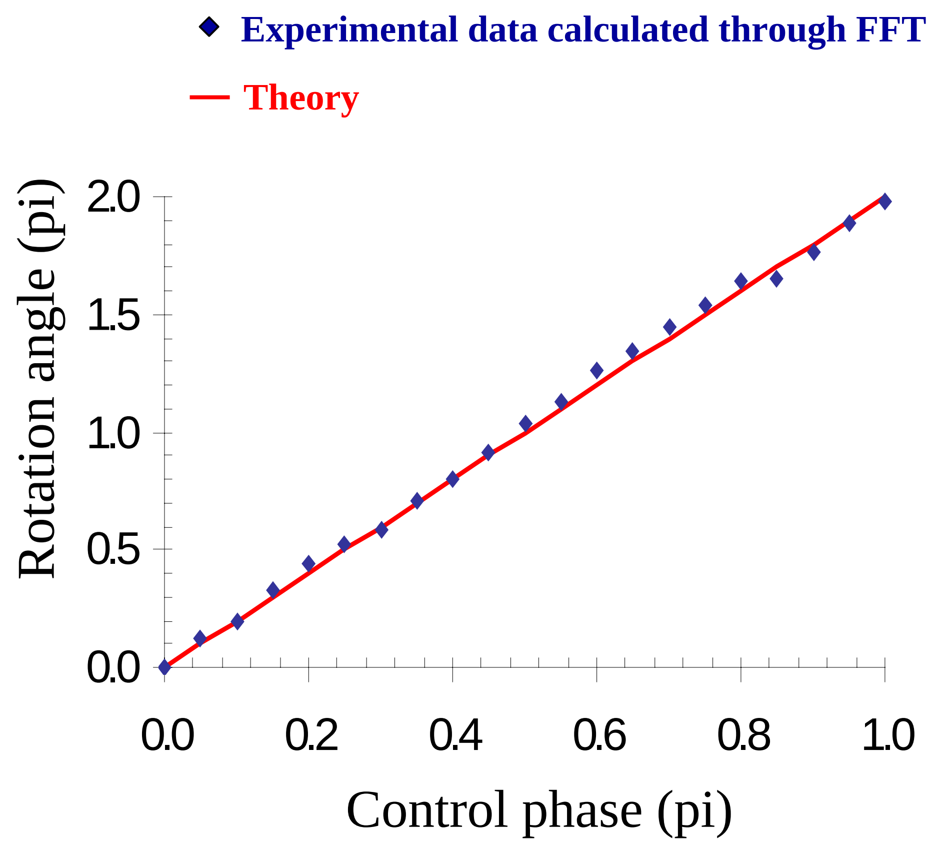
Experiment


Frequency stabilization to spectral hole
AWG
Photo diode
Ti: S Laser
AOM
Flip mirror
Tm+3:YAG
Photo diode
C1
C2
input
P1
P2
R
output
0
5
10
15
20
Time (μs)

Geometric phase shift and rotation angle



Phase shifted echo fields measured with heterodyne detection for phase shift varying from 0 ~ 2π, controlled by laser pulse of varying phase from 0 ~ π.



Bloch vector rotation angled measured using photon echo with measurement limited standard deviation of 0.03π.

Summary

- **Two basic Bloch vector rotations has been studied by means of laser controlled geometric phase changes.**
- **The control pulse sequences for the two basic geometric Bloch rotations have been designed.**
- **U_3 rotation has been demonstrated in thulium doped yttrium aluminum garnet (YAG).**
- **Geometric phase and rotation angles have been measured by heterodyned photon echo up to the accuracy of the measurement limit.**
- **U_2 rotation can be demonstrated in a similar way.**
- **Two-qubits CNOT gate be accomplished by pure geometric operation controlled by laser pulse sequence.**

Acknowledgments

•Krishna Rupavatharam, Spectrum Lab, Montana State U.

•Randy Reibel, Spectrum Lab, Montana State U.

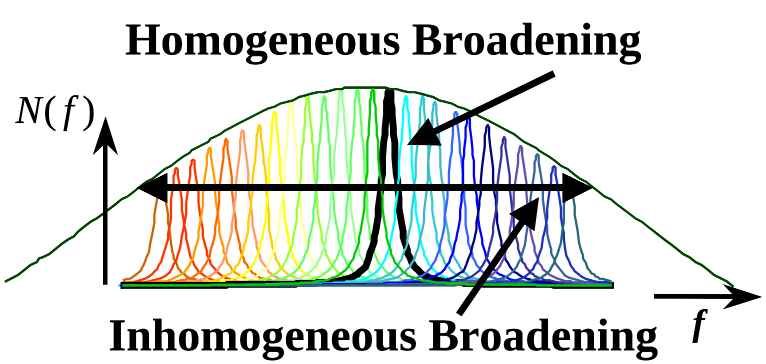
•Stefan Kroll, Lund Institute of Technology, Sweden
•Alexander Rebane, Montana State U.

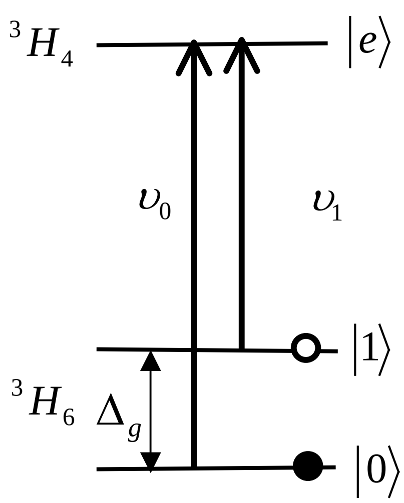
•Cone/Sun group, Montana State U.

•Sci. Mat. Inc. Bozeman, MT

•Air Force Office of Scientific Research grant (F49620-98-1-0283)

Physical qubit based on Tm doped crystals





Energy levels and Zeeman splitting

Properties of Tm:YAG at 4.2K

Frequency selectivity: Γ_{in}~20GHz,
Γ_h<100kHz
Coherence time: ~10μs for |e>,
~ms for |1>,
Lifetime: ~800μs for |e>,
~10s for |1>,
Oscillation strength: ~10^{-7} for $^3H_6$ -$^3H_4$ transition
Zeeman splitting: ~60MHz/Tesla for $^3H_6$
~20MHz/Tesla for $^3H_4$
Transition wavelength: 793nm

A physical qubit is:

- Represented by the atoms of identical energy level resonant at a selected frequency,
- Stored at hyperfine levels of the electronic ground state (1> and 0>) with coherence time of ~ms,
- Addressed selectively by laser pulse for quantum state initialization.
- Manipulated individually with quantum operation <μs

Multiple qubits and CNOT gate

Different qubits are labeled by their unique resonant frequencies, addressed individually by tuning excitation laser frequency to the channel of the selected qubits. Qubits can prepared by coherent and/or incoherent pummping through spectral hole burning process. Multiple frequency channels at one spatial spot favorites system scalability.Qubits are coupled by ion-ion interaction. CNOT gate can be realized by well-designed laser pulse sequence through either direct Rabi rotations or geometric rotations. The driving pulse sequence is independent of the qubits states.

Spectral partition for qubits